

INSTABILITIES OF CONDUCTING FLUID LAYERS IN CYLINDRICAL CELLS UNDER THE EXTERNAL FORCING OF WEAK MAGNETIC FIELDS

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In this work, we analyze recent results concerning the instabilities created in a layer of liquid metal by the action of time-dependent magnetic fields. The experimental setup allows the characterization of different patterns very close to the threshold. For very low frequencies of the forcing field, the axisymmetric fluid layer destabilizes with different azimuthal wavenumbers. An improved analysis allows the characterization of different patterns for interaction parameter values as low as $5 \cdot 10^{-3}$.

Introduction. It is very well known that the action of external magnetic fields can trigger different instabilities in conducting fluid layers [1]. These magnetic fields can be self-generated (dynamo action [2]) or externally applied. In the most generic situation, this magnetic field evolves in time, but depending on the frequency and electric conductivity of the fluid, the forces act on the bulk (low frequencies) or near the surface (high frequencies). Many industrial applications rely on this latter configuration, where high-frequency magnetic fields have allowed the development of special metallurgy procedures for certain materials [3]. On the other side, maybe because the potential applications are scarce, the domain of very low frequencies remains poorly studied. In this work, we will focus on a particular configuration, where a thin axisymmetric fluid layer with a free surface is forced through a time-dependent magnetic field parallel to the axis of the circular cavity. Due to the Lenz's law, the time-dependent magnetic field generates an azimuthal current that, being coupled with $\mathbf{B}$, produces a radial force. This force produces the destabilization of the static fluid layer, and a flow is created.

To our knowledge, there are very few experimental works on this field. Only the group of Y. Fautrelle in Grenoble has performed some measurements, but in a strongly non-linear regime [4, 5]. On the other hand, our system allows a much more precise threshold determination and spatial symmetry description, so we can compare with the theory that predicts an instability without threshold [6].

1. Experimental setup. The geometry of the experimental cell is a disc-shaped fluid layer placed inside a teflon cylindrical cavity with a diameter of 84 mm (see Fig. 1). The bottom part of this cell can be modified at will, so different configurations are possible. For the results that are presented here, this part has a small circular depression, so the fluid layer remains centered against the external magnetic field. Other configurations with an annular depression that yield a flattened doughnut-shape (with an internal hole) liquid layer has been also studied and will be presented elsewhere.

The fluid used is an In-Ga-Sn alloy, as a conductive layer thinner than 20 mm. An upper layer of hydrochloric acid (5%) was placed to prevent oxidation of the eutectic alloy (Fig. 2). The upper surface of the liquid metal remains as a mirror,

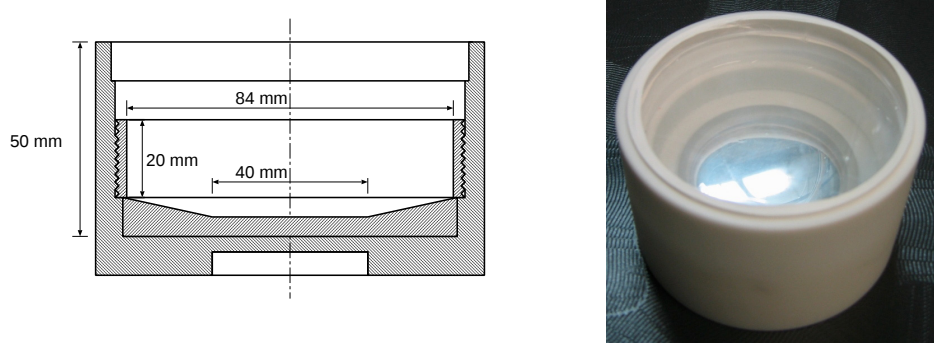


Fig. 1. Left: cross-section of the experimental cell. The bottom part can be replaced, so the fluid may adopt different shapes. Right: typical view of the experimental cell with a disc fluid layer immersed in a 2 cm deep hydrochloric acid layer, and the whole volume closed by a transparent cover. The metal liquid surface behaves like a mirror.

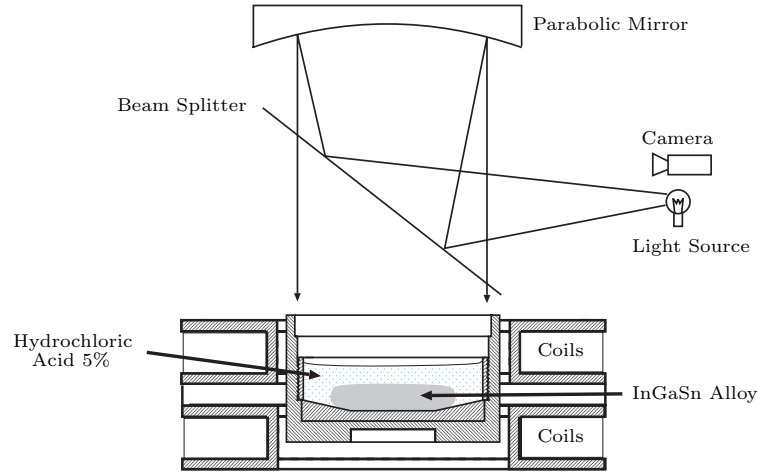


Fig. 2. Sketch of the observational apparatus. The parabolic mirror generates a parallel light beam from a point source, which reflects on the metal liquid surface. This light is recollected at the focal point of the mirror (in this graph, the camera and the light source are separated for clarity). Any deviation of this beam will appear in the camera as a bright or dark region, depending on the local curvature.

thus small deviations from the equilibrium position will deviate incident parallel beams and the deflection of the surface can be easily checked.

No external currents are applied on the problem, only an external magnetic field. This field evolves harmonically with frequencies up to 10 Hz, small enough to avoid skin depth effects. The magnitude of the magnetic field ranges from 0 to 0.1 T. Two coils placed outside the experimental cell generate the magnetic field, so the axis of this magnetic field is parallel to the axis of the cylinder (i.e. perpendicular to the free surface of the liquid metal layer). Two different ways are used to induce a time-dependent magnetic field. The first one is the modulation of the electric current circulating in the coils, the second one is moving the whole coil along its axis. A careful tracking and control of the displacement of the coils

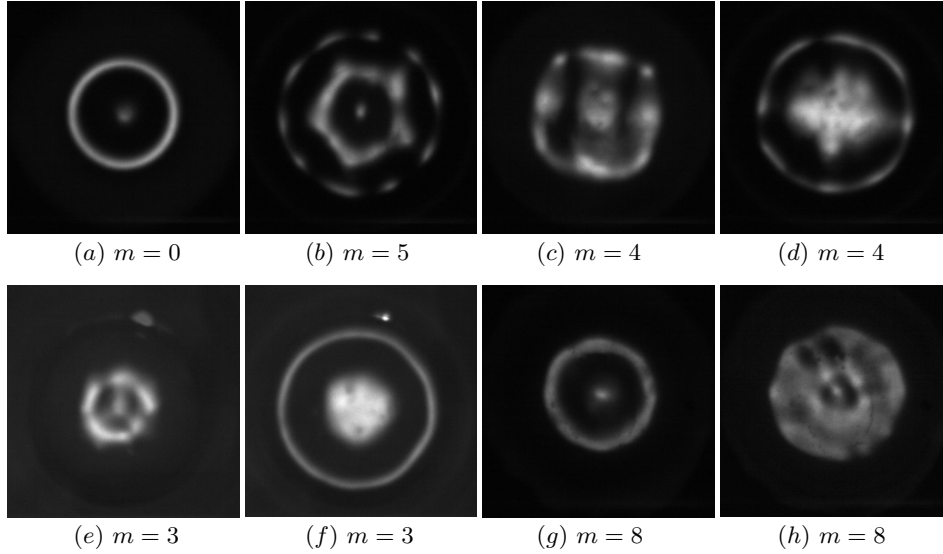


Fig. 3. Symmetries found for different experimental parameters. Explanation in the text.

is made in such a way that the magnetic field recorded in a given spatial position remains sinusoidal in time.

The observation setup is a variation of the method developed by Foucault [7] to verify the quality of parabolic mirrors, and allows the direct observation of the top view of the fluid layer. Any deflection can be easily detected, as deviations from the horizontal smaller than 50 cm can be determined. Using a camera connected to a computer, we can record the dynamic behaviour of these patterns. The data are recorded with a frequency of 25 Hz, large enough for the dynamics presented here, and the data are analysed using Matlab.

2. Results and discussion. In our setup, we control two experimental parameters: the frequency and the intensity of the magnetic field. We have checked the spatial patterns that appear for different regions, but here we will focus only on those that appear for frequencies smaller than 3.1 Hz and magnetic fields smaller than 0.064 T. For these values, the interaction parameter is smaller than $N < 0.2$ for all the presented results. An axisymmetric pattern appears without threshold and the mode $m = 0$ can be detected in all other patterns (Fig. 3a).

The previous work by Fautrelle and Sneyd predicts that each azimuthal wavenumber should live in different regions of the phase space (see [6]). In our system, we have found very strong resonances, which appear for certain frequencies, and regions, where different patterns are very close and can even coexist.

For example, in the case of a 55 mm diameter, 10 mm deep fluid layer, two narrow resonant regions were found around 1.53 Hz and 3.09 Hz. Depending on the intensity of the magnetic field, we may find (see snapshots in Figs. 3b–3h) modes $m = 3$ (1.55 Hz, 0.043 T), $m = 4$ (1.53 Hz, 0.064 T), $m = 5$ (3.09 Hz, 0.043 T), $m = 8$ (3.075 Hz, 0.043 T). However, a careful analysis of these patterns reveals that these patterns are not pure modes, because more than a single wavenumber appears simultaneously. Moreover, if we further increase the magnetic field intensity, a cycle is established between different patterns. This scenario is very different from the one described in [6]. Also, these patterns have been de-

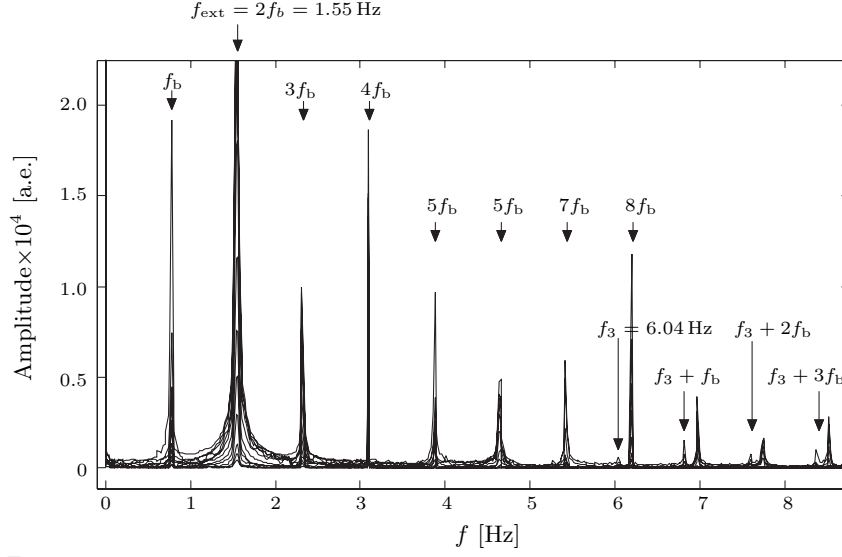


Fig. 4. Fourier transform of a time series of oscillations for a forcing of $f_0 = 1.55$ Hz at $B_0 = 0.035$ T. Many modes are excited simultaneously, but can be grouped in discrete sets associated with different spatial patterns.

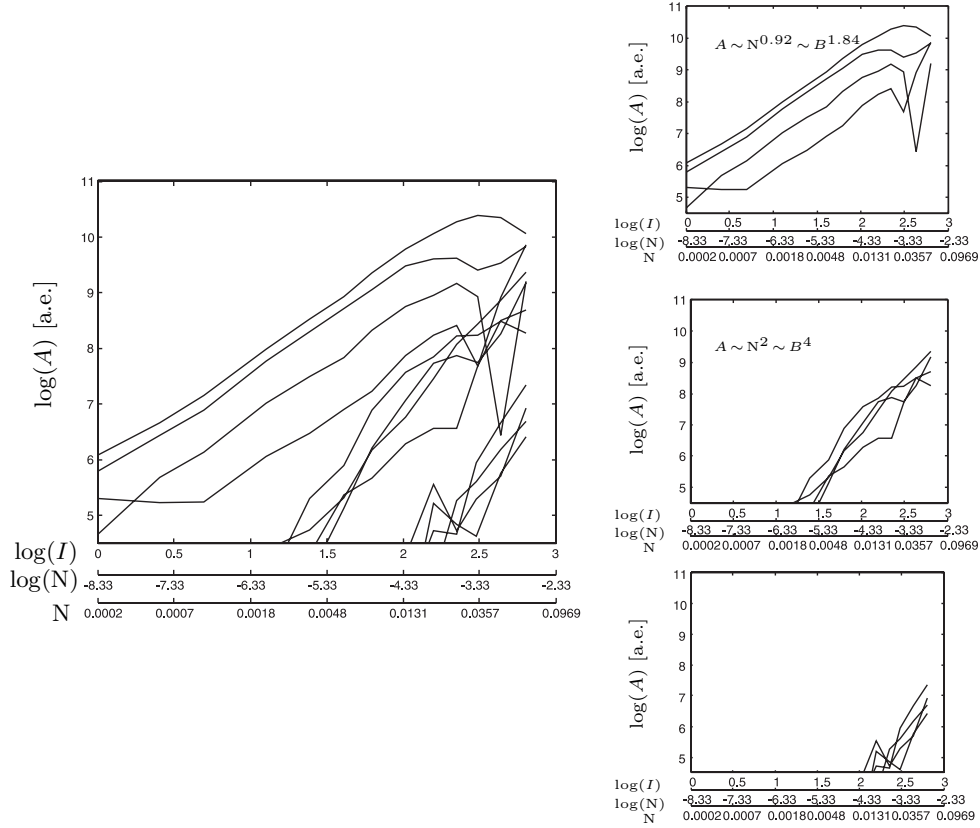


Fig. 5. Value of the maxima marked in Fig. 4, as a function of the interaction parameter. The main picture (left) has been split in three different patterns identified in real space (right, from top to bottom, axisymmetric, $m = 3$, $m = 4$).

tected for lower values if compared to those described in [4,5], where the surface is in a very strong non-linear regime.

We wanted to characterize the existence of these patterns as close to the threshold as possible, so we have optimized the data acquisition to record spatial and temporal evolutions for small interaction parameters (even for $N \sim 0.001$). Instead of merely identifying the patterns in a snapshot, we have performed a Fourier analysis of the temporal behaviour of these patterns (Fig. 4). From these data we can extract the harmonics that control the dynamics of the patterns. The relative amplitude of these peaks depend on the control parameters (mainly on the magnetic field intensity), and by recording their evolution, we can identify sets of harmonics, which evolve together.

This is the analysis that is performed in Fig. 5 for the peaks marked in Fig. 4. We can identify three different patterns, with different azimuthal wavenumbers and different power laws. One important point is that all these laws appear without threshold, and it seems that the only reason why these patterns are not detected for even smaller interaction parameters is the experimental sensitivity.

Once these patterns are determined, a 3D Fourier analysis is performed to filter each spatial pattern and determine the corresponding wavenumber. For the presented case, we have identified an axisymmetric pattern and non-axisymmetric structures with $m = 3$ and $m = 4$ modes. In Fig. 6, the original patterns have been split in three building blocks. The corresponding video reveals the dynamics of these patterns, which evolve with different frequencies.

A similar analysis has been made for other series, and this allowed us to decrease our threshold estimation by a factor of 4. So, the thick lines presented in the phase space (Fig. 7) correspond to an upper bound and can be even smaller. Actually, the power laws found in this analysis suggest that there is no threshold, as has been pointed out before. The amplitude of the different modes increases as a power of B for a given frequency, and although no threshold has been detected, different growing rates can be determined for different modes.

The final question is why the behaviour observed here is so different if compared to the scenario proposed by Fautrelle and Sneyd in [6]. A possible explanation is that in our problem the geometry is different, as there are two different interfaces, i.e. the top interface and the lateral wall of the disc. Each one of these walls can oscillate with different wave numbers, and so some kind of non-linear coupling is expected from these two effects that can reconnect the resonance win-

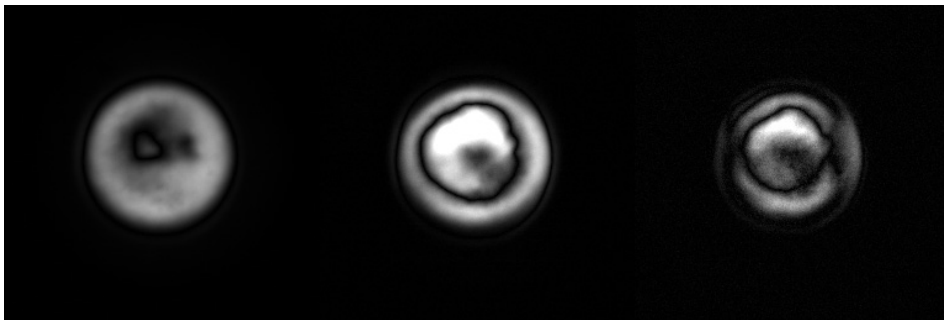


Fig. 6. Filtered patterns from the original signal. Axisymmetric (left), $m = 3$ (center) and $m = 4$ (right). These patterns are obtained through a 3D FFT analysis of the data, filtering around the respective modes. A video is available presenting the evolution of each pattern.

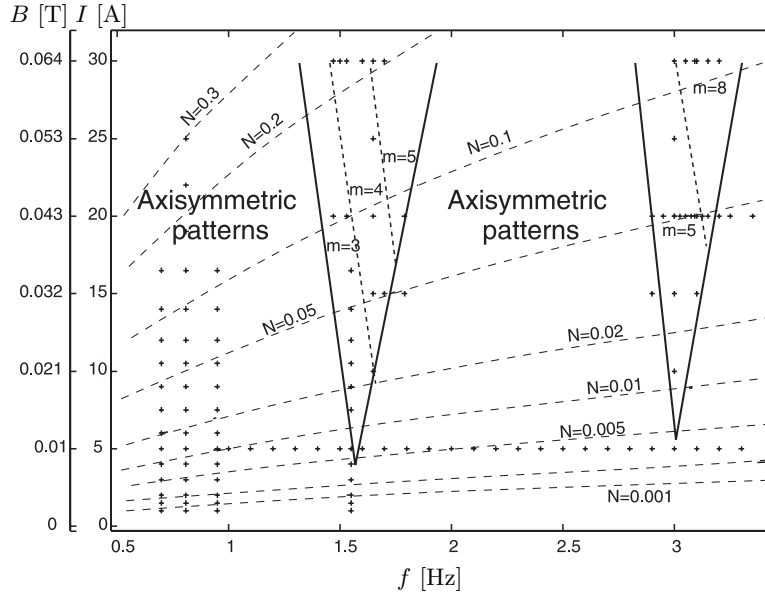


Fig. 7. Phase space of different patterns. Without threshold, an axisymmetric pattern appears for all frequencies. Other symmetries can be observed in restricted windows around discrete values (1.5 and 3 Hz). The size of these windows increases with the magnetic field intensity. The thick solid line, which separates the regions, where other symmetries are observed, is a minimum value, as it has been explained in the text. The same applies to the dashed lines, which mark the apparition of the different symmetries.

dows proposed by Fautrelle and Sneyd. A different setup is under construction to unlink both effects and check their separated influence on different instabilities.

3. Conclusion. We have presented a recent experimental work on the destabilization of a thin circular layer of a conducting fluid. Our experimental setup allows the characterization of the spatial patterns very close to any potential threshold, so we can work in a region, where the non-linearities are expected to be weak. Different symmetry breaking patterns (azimuthal wavenumbers $m = 0, 3, 4, 5, 8$) have been found, with a very rich dynamics (oscillations, alternating pattern). These patterns appear for parameter values in very restricted windows and for very small interaction parameters. Although the general characteristics of these windows are different from the predicted behaviour, a possible explanation for this discrepancy has been proposed.

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